

Quantum discord dynamics of two qubits in the single-mode cavities

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The dynamics of the quantum discord for two identical qubits in both two independent single-mode cavities and a common single-mode cavity are discussed. For the initial Bell state with correlated spins, while the entanglement sudden death can occur, the quantum discord vanishes only at discrete moments in the independent cavities and never vanishes in the common cavity. Interestingly, quantum discord and entanglement show opposite behaviors in the common cavity, unlike in the independent cavities. For the initial Bell state with anti-correlated spins, quantum discord and entanglement behave in the same way for both independent cavities and a common cavity. It is found that the detunings always stabilize the quantum discord.

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I. INTRODUCTION

Quantum entanglement, originated from nonlocal quantum correlation, is fundamental in quantum physics both for understanding the nonlocality of quantum mechanics [1] and plays an important role in almost all efficient protocols for quantum computations and communications [2]. Due to the unavoidable interaction with the environment, an initially entangled two-qubit system becomes totally disentangled after evolving for a finite time. This phenomena is called entanglement sudden death (ESD) [3] and has been recently demonstrated experimentally [4]. However, the entanglement may fail to capture the existence of the quantum correlation in some mixed separate states, in which the entanglement is considered not a good measure [5, 6].

Recently, a new kind of the quantum correlation, quantum discord (QD) has attracted a lot of attentions [7]. It provides the alternative route for measurement, which is present even under separable states [5]. The definition of the QD can be interpreted as the difference of the total quantum information of the two sub-systems A and B before and after the local operation on the one of them. The QD has been proved as a good measure of the non-classical correlations beyond entanglement. Furthermore, the QD has been indicated as the source to speed up the quantum computations [8, 9].

Some works have been devoted to the QD dynamics of two qubits coupled to Markovian [10, 11] and non-Markovian [12] environments. The comparisons with entanglement dynamics have been also performed. However, the relevant study on two two-level atoms (qubits) coupled to independent or common single-mode cavities without dissipations has not been found in the literature, to the best of our knowledge. We believe that the QD dynamics in these qubit systems is also of fundamen-

tal interest. In addition, some essential pictures can be clearly described and unfolded in the framework of the simple model where the exact solutions are available. Actually, the entanglement dynamics for two independent Jaynes-Cummings (JC) atoms has been well studied previously [13–18]. The ESD was observed obviously from the initial Bell states with correlated spins. This feature would prevent the application of the entanglement as basic resource for quantum information processing. What is the consequence for the QD in this kind of the qubit system?. It is just the main topic of the present study.

In the present paper, we will study the QD dynamics for two identical qubits in both two independent identical single-mode cavities and one common single-mode cavity. Comparisons with the corresponding pairwise entanglement, i.e. concurrence, are also given. The paper is organized as follows. In Sec. II and III, we derive the time dependent QD in these two systems if initiated from two typical Bell states. In Sec. IV, the results are given and discussions are made. The conclusion is presented in the last section.

II. QD IN TWO IDENTICAL JAYNES-CUMMINGS ATOMS

The Hamiltonian of two identical Jaynes-Cummings atoms is shown as

$$H_{JC} = \frac{\Delta}{2}(\sigma_z^A + \sigma_z^B) + \omega(a^\dagger a + b^\dagger b) + g(a^\dagger \sigma_-^A + \sigma_+^A a) + g(b^\dagger \sigma_-^B + \sigma_+^B b). \quad (1)$$

where $\sigma_k^{A(B)}$ ($k = x, y, z$) is the Pauli operator of the atom A(B), shown as $\sigma_z = |\uparrow\rangle\langle\uparrow| - |\downarrow\rangle\langle\downarrow|$ and $\sigma_x = |\uparrow\rangle\langle\downarrow| + |\downarrow\rangle\langle\uparrow|$, with $|\uparrow\rangle(|\downarrow\rangle)$ the excited (ground) state of the two-level atom, $a^\dagger(b^\dagger)$ and $a(b)$ are the creator and annihilator of the cavity A (B), respectively, Δ and ω are the frequencies of the atom and the cavity, g is the atom-cavity coupling strength. Here we set $\hbar = 1$ and the detuning $\delta = \Delta - \omega$.

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We first study the evolution of the QD initiated from the Bell state with anti-correlated spins, which has the following form

$$|\Psi_{\text{Bell}}^{(1)}\rangle = \sin \alpha |\downarrow\uparrow\rangle + \cos \alpha |\uparrow\downarrow\rangle. \quad (2)$$

Initially, the vacuum state of the cavity is considered, so the initial state of the whole system can be written as

$$\begin{aligned} |\Psi(0)\rangle &= (\sin \alpha |\downarrow\uparrow\rangle + \cos \alpha |\uparrow\downarrow\rangle) \otimes |00\rangle \\ &= \sin \alpha |\downarrow\uparrow 00\rangle + \cos \alpha |\uparrow\downarrow 00\rangle. \end{aligned} \quad (3)$$

The time dependent wave function can be generally expressed as [13]

$$\begin{aligned} |\Psi(t)\rangle &= x_1 |\uparrow\downarrow 00\rangle + x_2 |\downarrow\uparrow 00\rangle \\ &\quad + x_3 |\downarrow\downarrow 10\rangle + x_4 |\downarrow\downarrow 01\rangle, \end{aligned} \quad (4)$$

where the coefficients are

$$\begin{aligned} x_1 &= (Ae^{-i\lambda+t} + Be^{-i\lambda-t}) \cos \alpha \\ x_2 &= (Ae^{-i\lambda+t} + Be^{-i\lambda-t}) \sin \alpha \\ x_3 &= C(e^{-i\lambda+t} - e^{-i\lambda-t}) \cos \alpha \\ x_4 &= C(e^{-i\lambda+t} - e^{-i\lambda-t}) \sin \alpha. \end{aligned} \quad (5)$$

with the eigenfrequencies as

$$\lambda_{\pm} = \omega + \frac{\delta}{2} \pm \frac{\sqrt{\delta^2 + G^2}}{2}, \quad (6)$$

here $G = 2g$. The auxiliary parameters are shown as

$$\begin{aligned} A &= \frac{1}{2} \left(1 + \frac{\delta}{\sqrt{\delta^2 + G^2}} \right) \\ B &= \frac{1}{2} \left(1 - \frac{\delta}{\sqrt{\delta^2 + G^2}} \right) \\ C &= \frac{G}{2\sqrt{\delta^2 + G^2}}. \end{aligned} \quad (7)$$

The pairwise density matrix from Eq. (4) under the standard basis $\{|\downarrow\downarrow\rangle, |\downarrow\uparrow\rangle, |\uparrow\downarrow\rangle, |\uparrow\uparrow\rangle\}$ is thus expressed by tracing the freedoms of the cavities $\rho_{AB}(t) = \text{Tr}_{\text{cav}}\{\rho(t)\} = \text{Tr}_{\text{cav}}\{|\Psi(t)\rangle\langle\Psi(t)|\}$,

$$\rho_{AB}(t) = \frac{1}{4} \begin{pmatrix} |x_3|^2 + |x_4|^2 & 0 & 0 & 0 \\ 0 & |x_2|^2 & x_1^* x_2 & 0 \\ 0 & x_2^* x_1 & |x_1|^2 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}. \quad (8)$$

With this density matrix, the routine to derive the QD is formally given in the Appendix A. The von Neumann entropy for two atoms in Eq. (A2) is given by

$$\begin{aligned} S(A, B) &= -(|x_3|^2 + |x_4|^2) \log(|x_3|^2 + |x_4|^2) \\ &\quad - (|x_1|^2 + |x_2|^2) \log(|x_1|^2 + |x_2|^2), \end{aligned} \quad (9)$$

and the sub-system entropies in Eq. (A3) are shown as

$$\begin{aligned} S(A) &= -(1 - |x_1|^2) \log(1 - |x_1|^2) \\ &\quad - |x_1|^2 \log |x_1|^2, \end{aligned} \quad (10)$$

$$\begin{aligned} S(B) &= -(1 - |x_2|^2) \log(1 - |x_2|^2) \\ &\quad - |x_2|^2 \log |x_2|^2. \end{aligned} \quad (11)$$

From the Appendix A, one can find that the expressions of the elements in Eq. (A1) are

$$\begin{aligned} v_+ &= |x_3|^2 + |x_4|^2, \quad v_- = 0, \quad w = |x_2|^2, \\ x &= |x_1|^2, \quad y = x_1 x_2^*, \quad u = 0. \end{aligned} \quad (12)$$

Moreover,

$$\begin{aligned} X_{1,+} &= (|x_3|^2 + |x_4|^2) \cos^2 \theta + |x_2|^2 \sin^2 \theta, \\ X_{1,-} &= |x_1|^2 \cos^2 \theta, \\ |Y_1|^2 &= \frac{\sin^2 \theta}{4} |x_1|^2 |x_2|^2. \end{aligned}$$

and

$$\begin{aligned} X_{2,+} &= (|x_3|^2 + |x_4|^2) \sin^2 \theta + |x_2|^2 \cos^2 \theta, \\ X_{2,-} &= |x_1|^2 \sin^2 \theta, \\ |Y_2|^2 &= \frac{\sin^2 \theta}{4} |x_1|^2 |x_2|^2. \end{aligned}$$

Therefore these parameters are independent of ϕ . It follows that we can search the minimum of the conditional von Neumann entropy by only varying θ in the regime $[0, \pi/2]$. Following the procedures outlined in Appendix A, we can finally derive the quantum discord. Since α is limited to $(0, \pi/2)$, it can be numerically checked that $\theta = \pi/4$ corresponds to the minimum of the conditional entropy in the following calculations. The minimum of the conditional von Neumann entropy reads

$$S(A|\Pi^B) = - \sum_{\epsilon=\pm} \eta_{\epsilon} \log \eta_{\epsilon}, \quad (13)$$

where

$$\eta_{\pm} = \{1 \pm [(1 - 2|x_1|^2)^2 + 4|x_1 x_2|^2]^{1/2}\} / 2. \quad (14)$$

As a result, the quantum discord is finally given by

$$\begin{aligned} \mathcal{D} &= -(1 - |x_2|^2) \log(1 - |x_2|^2) - |x_2|^2 \log |x_2|^2 \\ &\quad + (|x_3|^2 + |x_4|^2) \log(|x_3|^2 + |x_4|^2) \\ &\quad + (|x_1|^2 + |x_2|^2) \log(|x_1|^2 + |x_2|^2) \\ &\quad - \sum_{\epsilon=\pm} \eta_{\epsilon} \log \eta_{\epsilon}. \end{aligned} \quad (15)$$

For later use, we also list the expression for concurrence derived in Ref. [13] as

$$C_{AB}(t) = |\sin 2\alpha| [1 - 4C^2 \sin^2(\sqrt{\delta^2 + G^2}t/2)]. \quad (16)$$

Next, we consider the Bell state with correlated spin as the initial atomic state, which is

$$|\Psi_{\text{Bell}}^{(2)}\rangle = \sin \alpha |\downarrow\downarrow\rangle + \cos \alpha |\uparrow\uparrow\rangle. \quad (17)$$

Including the initial vacuum cavities, the wave function of the whole system can be expressed as

$$\begin{aligned} |\Psi\rangle(t) &= x_1 |\uparrow\uparrow 00\rangle + x_2 |\downarrow\downarrow 11\rangle + x_3 |\uparrow\downarrow 01\rangle \\ &\quad + x_4 |\downarrow\uparrow 10\rangle + x_5 |\downarrow\downarrow 00\rangle, \end{aligned} \quad (18)$$

where the coefficients are

$$\begin{aligned} x_1 &= (Ae^{-i\lambda+t} + Be^{-i\lambda-t})^2 \cos \alpha, \\ x_2 &= AB(e^{-i\lambda+t} - e^{-i\lambda-t})^2 \cos \alpha, \\ x_3 &= C(e^{-i\lambda+t} - e^{-i\lambda-t})(Ae^{-i\lambda+t} + Be^{-i\lambda-t}) \cos \alpha, \\ x_4 &= C(e^{-i\lambda+t} - e^{-i\lambda-t})(Ae^{-i\lambda+t} + Be^{-i\lambda-t}) \cos \alpha, \\ x_5 &= \sin \alpha, \end{aligned} \quad (19)$$

The eigenfrequencies and the auxiliary parameters are the same as those in Eqs. (6) and (7). Then under the standard basis $\{|\downarrow\downarrow\rangle, |\downarrow\uparrow\rangle, |\uparrow\downarrow\rangle, |\uparrow\uparrow\rangle\}$, the pairwise density matrix is shown as

$$\rho_{AB}(t) = \begin{pmatrix} |x_2|^2 + |x_5|^2 & 0 & 0 & x_1^* x_5 \\ 0 & |x_4|^2 & 0 & 0 \\ 0 & 0 & |x_3|^2 & 0 \\ x_1 x_5^* & 0 & 0 & |x_1|^2 \end{pmatrix}. \quad (20)$$

Hence, the joint von Neumann entropy is derive as

$$\begin{aligned} S(A, B) &= -|x_3|^2 \log |x_3|^2 - |x_4|^2 \log |x_4|^2 \\ &\quad - \sum_{\epsilon=\pm} \Omega_\epsilon \log \Omega_\epsilon, \end{aligned} \quad (21)$$

where

$$\begin{aligned} \Omega_\pm &= \{(|x_1|^2 + |x_2|^2 + |x_5|^2) \\ &\quad \pm \sqrt{(|x_2|^2 + |x_5|^2 - |x_1|^2)^2 + 4|x_1|^2|x_5|^2}\}/2. \end{aligned} \quad (22)$$

And the sub-system entropy can be derive as

$$\begin{aligned} S(A) &= -(|x_2|^2 + |x_4|^2 + |x_5|^2) \log(|x_2|^2 + |x_4|^2 + |x_5|^2) \\ &\quad - (|x_1|^2 + |x_3|^2) \log(|x_1|^2 + |x_3|^2), \end{aligned} \quad (23)$$

$$\begin{aligned} S(B) &= -(|x_2|^2 + |x_3|^2 + |x_5|^2) \log(|x_2|^2 + |x_3|^2 + |x_5|^2) \\ &\quad - (|x_1|^2 + |x_4|^2) \log(|x_1|^2 + |x_4|^2). \end{aligned} \quad (24)$$

Similar to the above Bell state with anti-correlated spins, if we focus on $\alpha \in (0, \pi/2)$, $\theta = \pi/4$ also corresponds to the minimum of the conditional von Neumann entropy. Hence, the minimum of the conditional entropy is given by

$$S(A|\Pi^B) = - \sum_{\epsilon=\pm} \eta_\epsilon \log \eta_\epsilon, \quad (25)$$

where

$$\eta_\pm = \{1 \pm \sqrt{(1 - 2|x_1|^2 - 2|x_3|^2)^2 + 4|x_1|^2|x_5|^2}\}/2. \quad (26)$$

As a result, the quantum discord can be derived from Eqs. (23), (24), and (25) as

$$\mathcal{D} = S(B) - S(A, B) + S(A|\Pi^B). \quad (27)$$

The concurrence in this case has been also derived previously [13], and is also collected here

$$C_{AB}(t) = \max\{0, f(t)\}$$

$$\begin{aligned} f(t) &= [1 - 4C^2 \sin^2(\sqrt{\delta^2 + G^2 t}/2)] \\ &\quad [|\sin 2\alpha| - 8C^2 \sin^2(\sqrt{\delta^2 + G^2 t}/2) \cos^2 \alpha]. \end{aligned} \quad (28)$$

Specially at resonance ($\delta = 0$), the entanglement sudden transition occurs only for $\alpha < \alpha_c$, where $\alpha_c = \pi/4$.

III. QD IN TWO IDENTICAL QUBITS IN ONE COMMON SINGLE-MODE CAVITY

The Hamiltonian of two identical qubits interacting with one common single-mode cavity reads

$$H_{\text{DN2}} = \frac{\Delta}{2}(\sigma_z^A + \sigma_z^B) + \omega(a^\dagger a + \frac{1}{2}) + g \sum_{k=A,B} (a\sigma_k^+ + \sigma_k a^\dagger).$$

where a^\dagger and a are the creator and annihilator of the common cavity. Actually, it is just the $N = 2$ Dicke model [19]. The detunning is also set as $\delta = \Delta - \omega$.

If the initial atom state is selected as the Bell state with anti-correlated spins, we can obtain the time dependent wavefunction as

$$|\Psi(t)\rangle = x_1|\uparrow\downarrow 0\rangle + x_2|\downarrow\uparrow 0\rangle + x_3|\downarrow\downarrow 1\rangle, \quad (29)$$

with

$$\begin{aligned} x_1 &= \langle \uparrow\downarrow 0 | e^{-iH_{\text{DN2}}t} | \Psi_{\text{Bell}}^{(1)} \rangle, \\ x_2 &= \langle \downarrow\uparrow 0 | e^{-iH_{\text{DN2}}t} | \Psi_{\text{Bell}}^{(1)} \rangle, \\ x_3 &= \langle \downarrow\downarrow 1 | e^{-iH_{\text{DN2}}t} | \Psi_{\text{Bell}}^{(1)} \rangle. \end{aligned}$$

Then the pairwise density matrix under standard basis $\{|\downarrow\downarrow\rangle, |\downarrow\uparrow\rangle, |\uparrow\downarrow\rangle, |\uparrow\uparrow\rangle\}$ is given by

$$\rho_{AB}(t) = \begin{pmatrix} |x_3|^2 & 0 & 0 & 0 \\ 0 & |x_2|^2 & x_1^* x_2 & 0 \\ 0 & x_1 x_2^* & |x_1|^2 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}. \quad (30)$$

For the resonant case ($\delta = 0$), we specify the coefficients of the wavefunction in Eq. (29) as

$$\begin{aligned} x_1 &= \frac{\cos \alpha}{2}(\cos \sqrt{2}\lambda t + 1) + \frac{\sin \alpha}{2}(\cos \sqrt{2}\lambda t - 1), \\ x_2 &= \frac{\cos \alpha}{2}(\cos \sqrt{2}\lambda t - 1) + \frac{\sin \alpha}{2}(\cos \sqrt{2}\lambda t + 1), \\ x_3 &= \frac{-i}{\sqrt{2}}(\cos \alpha + \sin \alpha) \sin \sqrt{2}\lambda t. \end{aligned}$$

After the numerical checks, we find that $\theta = \pi/4$ corresponds to the minimum of the conditional entropy at arbitrary time. Hence the QD is described as

$$\begin{aligned} \mathcal{D} &= -(1 - |x_2|^2) \log(1 - |x_2|^2) - |x_2|^2 \log |x_2|^2 \\ &\quad + (|x_3|^2) \log(|x_3|^2) + (1 - |x_3|^2) \log(1 - |x_3|^2) \\ &\quad - \sum_{\epsilon=\pm} \eta_\epsilon \log \eta_\epsilon, \end{aligned} \quad (31)$$

with

$$\eta_\pm = \{1 \pm [(1 - 2|x_1|^2)^2 + 4|x_1 x_2|^2]^{1/2}\}/2. \quad (32)$$

Besides, the concurrence of the two atoms can also be given as

$$C_{AB}(t) = 2 \max\{0, |x_1 x_2|\}. \quad (33)$$

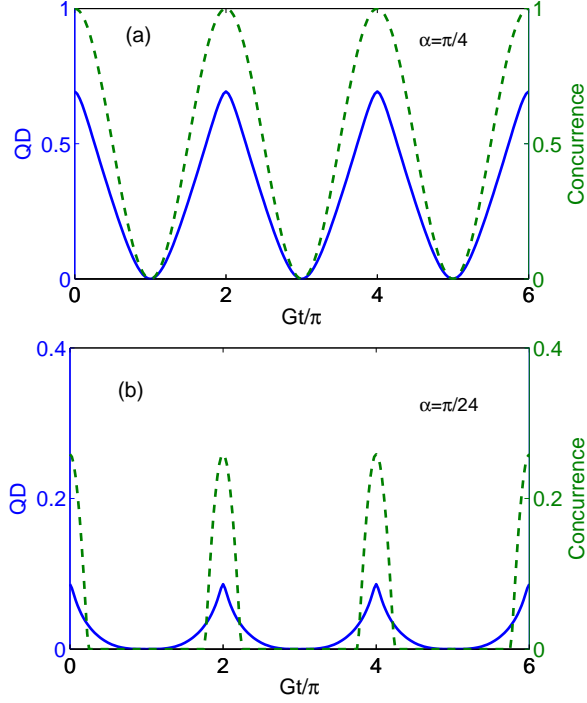


FIG. 1: (Color online) Resonant dynamics of QD and concurrence of two identical JC atoms with the initial atomic Bell states with anti-correlated spins (a) and correlated spins (b). $\omega = 1.0$. The blue solid line and the green dashed line are corresponding to QD and concurrence respectively.

If starting from the initial Bell state with correlated spins, the wavefunction can be expressed as

$$|\Psi(t)\rangle = x_1|\uparrow\uparrow 0\rangle + x_2|\downarrow\downarrow 2\rangle + x_3|\uparrow\downarrow 1\rangle + x_4|\downarrow\uparrow 1\rangle + x_5|\downarrow\downarrow 0\rangle. \quad (34)$$

Hence the pairwise density matrix can be derived as

$$\rho_{AB}(t) = \begin{pmatrix} |x_2|^2 + |x_5|^2 & 0 & 0 & x_1^* x_5 \\ 0 & |x_4|^2 & x_3^* x_4 & 0 \\ 0 & x_3 x_4^* & |x_3|^2 & 0 \\ x_1 x_5^* & 0 & 0 & |x_1|^2 \end{pmatrix}. \quad (35)$$

The coefficients at resonant condition are shown as

$$\begin{aligned} x_1 &= \frac{\cos \alpha}{3} e^{-i\omega t} (2 + \cos \sqrt{6}\lambda t), \\ x_2 &= \frac{\sqrt{2} \cos \alpha}{3} e^{-i\omega t} (\cos \sqrt{6}\lambda t - 1), \\ x_3 &= \frac{-i \cos \alpha}{\sqrt{6}} e^{-i\omega t} \sin \sqrt{6}\lambda t, \\ x_4 &= \frac{-i \cos \alpha}{\sqrt{6}} e^{-i\omega t} \sin \sqrt{6}\lambda t, \\ x_5 &= \sin \alpha. \end{aligned}$$

Then we can obtain the QD numerically.

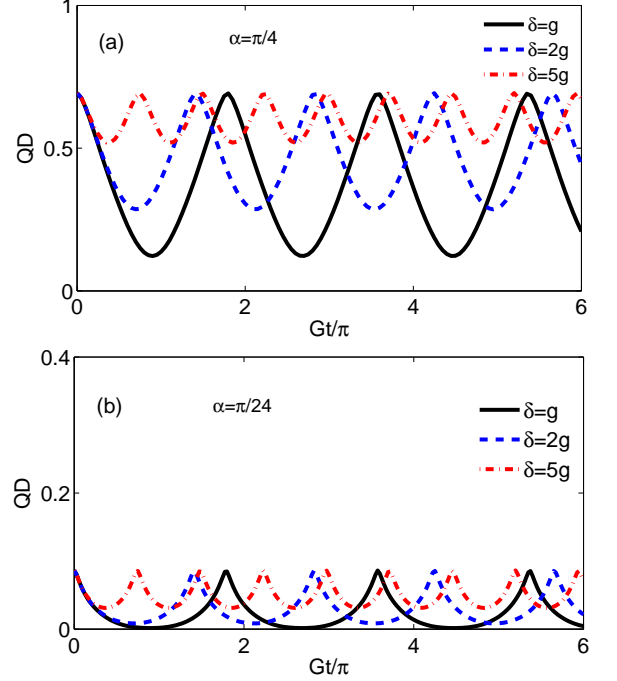


FIG. 2: (Color online) Off-resonant dynamics of QD for two identical JC atoms with the initial atomic Bell states with anti-correlated spins (a) and correlated spins (b) for different detunings $\delta = g, 2g$, and $5g$. $\omega = 1.0$.

While for the concurrence at resonance, we know that the pairwise density matrix in Eq. (35) has the form as

$$\rho_{AB}(t) = \begin{pmatrix} v_+ & 0 & 0 & u^* \\ 0 & y & y & 0 \\ 0 & y & y & 0 \\ u & 0 & 0 & v_- \end{pmatrix}. \quad (36)$$

Then we can derive it as

$$C_{AB}(t) = 2 \max\{0, |x_1 x_5| - |x_3|^2, |x_3|^2 - |x_1| \sqrt{|x_2|^2 + |x_5|^2}\}. \quad (37)$$

From the definition, we find that there exists a critical bound for α . The ESD happens only for $\alpha < \alpha_c$. The α_c is determined by

$$\tan^2 \alpha_c - 4 \tan \alpha_c + 1 = 0, \quad (38)$$

resulting in $\alpha_c = \arctan(2 - \sqrt{3}) = \pi/12$.

IV. RESULTS AND DISCUSSIONS

First, we compare the QD with the concurrence in the two identical JC atoms with two initial atomic states, i. e. the Bell states with anti-correlated spins and correlated spins, for zero detunings. The results are collected in Fig. 1. The evolution of both QD and concurrence for

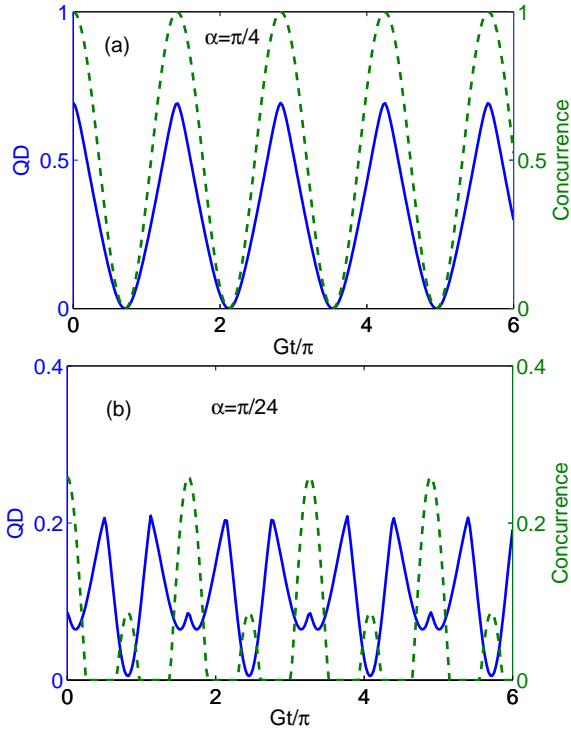


FIG. 3: (Color online) Resonant dynamics of QD and concurrence of two qubits coupled to a common cavity with the initial atomic Bell states with anti-correlated spins (a) and correlated spins (b). $\omega = 1.0$. The blue solid line and the green dashed line are corresponding to QD and concurrence respectively.

the initial Bell state with anti-correlated spins display similar behavior, as seen in Fig. 1(a). Yonac *et al.* [13] has shown that the ESD only occurs in the initial atomic Bell states with correlated spin, where the entanglement can fall abruptly to zero and vanish for a period of time before revival. It is interesting to note from Fig. 1(b) that during the period of ESD, QD becomes small but is always finite, except vanish at discrete moments $t = (2k + 1)\pi/G$, ($k = 0, 1, 2, \dots$).

Then, we show the effects of the detunings on the QD in independent cavities in Fig. 2, starting from these two Bell states. Interestingly, the amplitude of oscillation of the QD as a function of time is suppressed monotonically by the detunings for both initial Bell stats. More importantly, the zeros of the QD at discrete instants shown in Fig. 1(b) disappear with the finite detunings.

Next, we compare the QD with the concurrence in the two identical qubits coupled to the common cavity with two initial atomic Bell states. The results for zero detuning are presented in Fig. 3. For the initial atomic Bell state with anti-correlated spins, similar behaviors for both QD and concurrence are observed. For the initial atomic Bell state with correlated spins, one can find from Fig. 3(b) that the ESD can occur, but the QD never vanishes. Interestingly, QD and entanglement show opposite

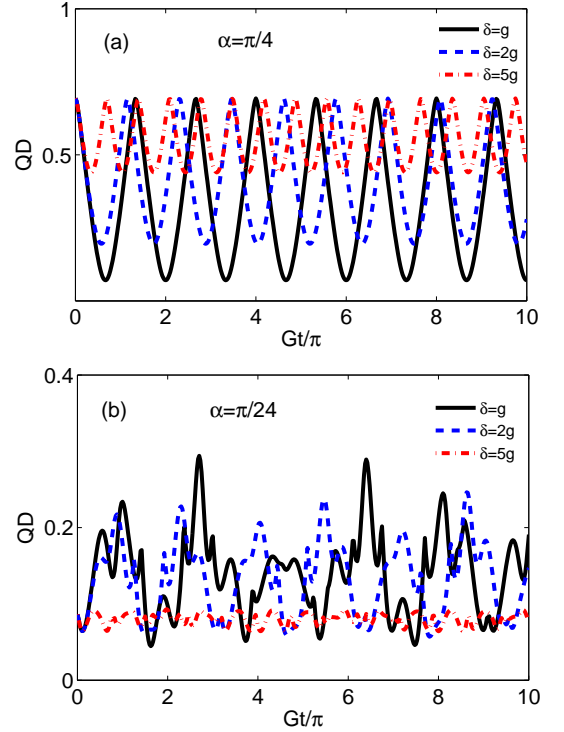


FIG. 4: (Color online) Off-resonant dynamics of QD for two qubits coupled to a common cavity with the initial atomic Bell states with anti-correlated spins (a) and correlated spins (b) for different detunings $\delta = g, 2g$, and $5g$. $\omega = 1.0$.

behaviors. Especially, during the period of ESD, the QD always becomes larger, in sharp contrast with that observed in the independent cavities (c.f. Fig. 1(b)).

Note that the critical parameter $\alpha_c = \pi/12$ below which the ESD can occur in the common cavity is smaller than $\alpha_c = \pi/4$ in independent cavities. The instant vanish of QD is absent in the common cavity, implying that the common cavity enhance the QD. So, it is suggested that the quantum correlation in the common cavity is stronger than that in independent cavities in some sense.

The effect of the detunings on the QD of two qubits coupled to a common cavity is also studied. As shown in Fig. 4(a), for the initial Bell state with anti-correlated spins, the amplitude of oscillation of the QD as a function of time is also suppressed monotonically by the detunings and little bit larger than that in two cavities (c.f. Fig. 2(a)). While for the initial atomic Bell state with correlated spins with $\alpha = \pi/24$ where ESD can occur, the oscillation of the QD for two qubits in the common cavity is suppressed considerably with detunings, as shown in Fig. 4(b). Especially, for large detunings $\delta = 5g$, the QD remains almost unchanged. In this case, we find that the components of the Bell stats show slightly variation with time for large detunning, due to the fact that large detunings prevent the hopping for photons between different atomic levels to certain degree.

V. CONCLUSIONS

In this paper, the QD dynamics of two qubits in both independent and common cavities are investigated. The comparisons with the entanglement evolution are also performed. For the initial atomic Bell state with anti-correlated spins, the QD and entanglement show the similar behaviors for both cavities. But for the initial atomic Bell state with correlated spins, the QD and entanglement behave in a remarkably different way. The ESD may occur for both cavities, but the QD never vanishes suddenly. For the independent cavities, the QD vanishes only at discrete instants and can be lifted with finite detunings. In the common cavity the QD is always finite. Especially, the QD and entanglement display an opposite behavior in the common cavity, different from those in independent cavities. The detunings play important role on the QD dynamics. It always stabilizes the QD, which could be helpful in the real applications of the QD as the better resource in quantum information science and quantum computing.

VI. ACKNOWLEDGEMENT

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Appendix A: Derivation of the quantum correlation

In the present paper, the general pairwise density matrix under the standard basis $\{|\downarrow\uparrow\rangle, |\downarrow\downarrow\rangle, |\uparrow\downarrow\rangle, |\uparrow\uparrow\rangle\}$, is shown as

$$\rho_{AB}(t) = \begin{pmatrix} v_+ & 0 & 0 & u^* \\ 0 & w & y^* & 0 \\ 0 & y & x & 0 \\ u & 0 & 0 & v_- \end{pmatrix}. \quad (\text{A1})$$

The von Neumann entropy of the two atoms is

$$S(A, B) = \sum_{\epsilon=\pm, k=1,2} \Omega_{k,\epsilon} \log \Omega_{k,\epsilon}, \quad (\text{A2})$$

with

$$\Omega_{1,\pm} = \frac{(v_+ + v_-) \pm \sqrt{(v_+ - v_-)^2 + 4|u|^2}}{2},$$

$$\Omega_{2,\pm} = \frac{(w + x) \pm \sqrt{(w - x)^2 + 4|y|^2}}{2}.$$

The reduced sub-system density matrices for A and B are obtained as

$$\rho_A(t) = (v_+ + w)|\downarrow\rangle_A \langle\downarrow| + (x + v_-)|\uparrow\rangle_A \langle\uparrow|,$$

$$\rho_B(t) = (v_+ + x)|\downarrow\rangle_B \langle\downarrow| + (w + v_-)|\uparrow\rangle_B \langle\uparrow|.$$

Hence, we derive the corresponding von Neumann entropies as

$$S(A) = -(v_+ + w) \log(v_+ + w) - (x + v_-) \log(x + v_-), \quad (\text{A3})$$

$$S(B) = -(v_+ + x) \log(v_+ + x) - (w + v_-) \log(w + v_-),$$

While for the conditional density matrix $\rho_{A|\Pi^B}$, projection basis are considered as

$$|\Phi_1\rangle_B = \cos\theta |\downarrow\rangle_B + e^{i\phi} \sin\theta |\uparrow\rangle_B,$$

$$|\Phi_2\rangle_B = e^{-i\phi} \sin\theta |\downarrow\rangle_B - \cos\theta |\uparrow\rangle_B.$$

The conditional density operator is expressed as

$$\rho_{A|\Pi_k^B} = \Pi_k^B \rho_{AB} \Pi_k^B / p_k,$$

where $\Pi_k^B = \mathbf{I}_A \otimes |\Phi_k\rangle_B \langle\Phi_k|$ and $p_k = \text{Tr}_{AB}\{\rho_{A|\Pi_k^B}\}$. Specifically under the projections in Eq. (A4),

$$\rho_{A|\Pi_k^B} = |\Phi_k\rangle_B \langle\Phi_k| \otimes \{|\downarrow\rangle_A \langle\downarrow| X_{k,+} + |\uparrow\rangle_A \langle\uparrow| X_{k,-} + |\downarrow\rangle_A \langle\uparrow| Y_k + |\uparrow\rangle_A \langle\downarrow| Y_k^*\} / p_k.$$

For $k = 1$, we show

$$X_{1,+} = v_+ \cos^2\theta + w \sin^2\theta,$$

$$X_{1,-} = x \cos^2\theta + v_- \sin^2\theta,$$

$$Y_1 = (y^* e^{-i\phi} + u^* e^{i\phi}) \sin\theta \cos\theta.$$

For $k = 2$,

$$X_{2,+} = v_+ \sin^2\theta + w \cos^2\theta,$$

$$X_{2,-} = x \sin^2\theta + v_- \cos^2\theta,$$

$$Y_2 = -(y^* e^{-i\phi} + u^* e^{i\phi}) \sin\theta \cos\theta.$$

Then the eigenvalues of the conditional density matrix reads

$$\eta_{k,\pm} = \frac{1}{2p_k} \{ (X_{k,+} + X_{k,-}) \pm [(X_{k,+} - X_{k,-})^2 + 4|Y_k|^2]^{1/2} \}. \quad (\text{A4})$$

The conditional von Neumann entropy is described as

$$S(A|\Pi^B) = \sum_{k=1,2} -p_k \text{Tr}_A \{ \rho_{A|\Pi_k^B} \log \rho_{A|\Pi_k^B} \} \quad (\text{A5})$$

$$= - \sum_{\epsilon=\pm, k=1,2} \sum p_k \eta_{k,\epsilon}(\theta, \phi) \log \eta_{k,\epsilon}(\theta, \phi).$$

As a result, the quantum discord can be obtained by

$$\mathcal{D} = \min\{S(B) - S(A, B) + S(A|\Pi^B)\},$$

and the classical correlation is given as

$$\mathcal{C} = \max\{S(A) - S(A|\Pi^B)\}.$$

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